Analele Universității De Vest Din Timişoara
Vol. XLVI, 2005
Seria Fizică

PHYSICAL MODEL FOR TSUNAMIS WAVES*

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Abstract. The most representative studies about sea waves offer only factual cinematic descriptions of these impressive phenomena. Our work tries to eliminate the information poverty concerning to the tsunamis as a transversal wave motion. Our dynamical model précised the characteristics of a tsunamis wave produced by some shallow submarine earthquake and volcanic eruption. Started from the local variation of hydrostatic pressure and from the fact that the restoring forces have a particular quasielastic character we stated the period, equation, the speed and wavelength of this kind of water waves versus the depth of ocean floor perturbation The evaluated characteristics are in good accord with those observed for the catastrophic tsunamis, from 12/26/2004, caused by undersea Sumatra's earthquake of Richter magnitude greater than 9 deg. and epicenter depth of less than 40 kilometers.
Keywords: Tsunamis waves, equations, characteristics.

1. Introduction.

Ordinary waves the most extended phenomena visible at the surface of planetary ocean are undulatory phenomena of low depth produced by the wind. . Even in the case of billow, the great waves, caused by a violent tempests or storm the depth of water movement not exceed a hundred meters. The huge sea waves, named today tsunamis, are determined in the seawater by the major short duration disturbance of the ocean floor [1]. All these traveling motions visible on the surface of the water in a form of a moving ridge or swell is caused, to a great extent, by hydrostatical pressure and to a low extent by superficial tension. The earliest observations and attempts to explain, the tsunamis waves were made by Greek geographer Strabo whose contribution came closest to modern conceptions about these deluges that are caused by upheavals of the bed of the sea. The most representative studies about tsunami and tidal waves show us only factologic descriptions of these impressive phenomena [2,3]. Our work tries to eliminate this information poverty by an operational, dynamical model of the tsunamis wave as a solitar transversal progressive motion.

* Oral presentation, TIM-05 conference, 24-25 November, 2005, Timisoara

2. Period of seawater oscillations

Period of freely vibrating system or natural mode are determined by the quasielastic character of the restoring forces; the properties of the medium through which a certain disturbance exist determine the speed of propagation of this or the wave front velocity. The equation, $T=2\pi(m/k)^{1/2}$, of the period of motion, the time taken for one complete oscillation of given mass is very important. It applied any system moving under Hooke's law force provided the mass moved and restoring force can be simple related through $F=ma$. We are familiar with the fact that in the case of a floating rigid body when the Archimedes' force buoyed up the body, or in the case of water placed in glass U-tube, any vertical disturbance shall produce a periodic motion because the hydrostatic pressure force at the level of water is quasielastic one.

Returning now to the case of the periodic motion of seawater caused by any large-scale, short-duration disturbance of the ocean floor, in figure 1 we can see that any vertical displacement, of a water mass, m, will determine by its weight a restoring force:

$$F_z = mg = V\rho g = Sz\rho g = -kz \quad \text{where } k = S\rho g$$

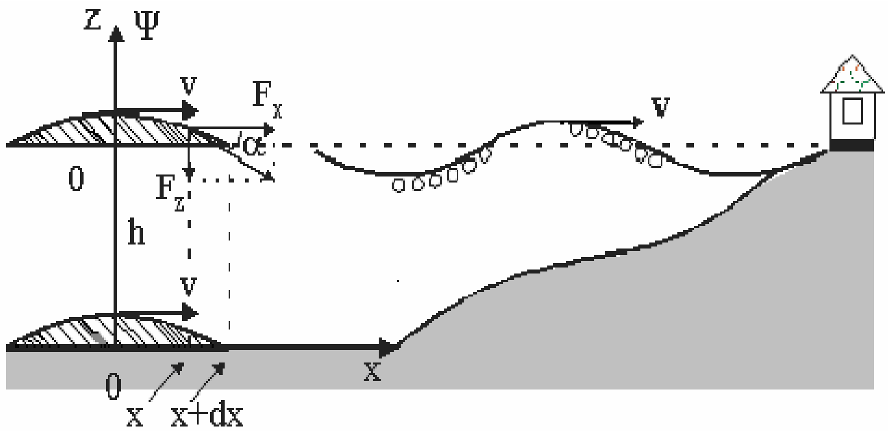

Fig. 1. A disturbance in ocean floor generates a transverse tsunami wave in seawater

This quasielastic force that acting on the entier mass $M=Sh\rho$ of water column of h depth and S section will cause a particular vibratory motion determined by the character of hidrostaic presure to act equaly in every direction. Neglecting the compression, viscosity, surface tension and the change in the mass of water column, the dynamic equation of this motion (Newton's law) will be $M\frac{d^2z}{dt^2} = -\rho gAz$, giving, for the angular frequency, the relationship $\omega = \sqrt{\frac{\rho gA}{M}}$

From this, the period of oscillations of the whole water column and of each water particle will be:

$$T = 2\pi\sqrt{\frac{M}{k}} = 2\pi\sqrt{\frac{Ah\rho}{A\rho g}} = 2\pi\sqrt{\frac{h}{g}} = \frac{2\pi}{g}\sqrt{\frac{p}{\rho}} \qquad (1)$$

Note the similarity to the period of water oscillations in an U-tube, but also the subtle difference that a free water column has the same period as a confined cylindrical water's column of length *2h.* The relation of the water' bodily oscillation to the location of considered disturbance is in fact directly connected to the depth or more exactly to the local hydrostatic pressure $p = \rho gh$. If, for example, the depth of the sea floor is 1 km, where the hydrostatic pressure is about 100 atm., the period of such vertical oscillations would be more than 1 min. The real period of free oscillations with damping have to be grater, and the motion become a periodically waves pulse. It will be noted that equation (1) has not any connection with those of longitudinal oscillations caused by the restoring compressional elastic forces. From the foregoing analysis result that the real oscillatory motion in water's volume is determined by the special characteristic of hydrostatic pressure that act in a horizontal plane with the same values, but increasing with the seawater depth..

3. Equation and speed of tsunami waves

Suppose now that we have an any large-scale, short-duration disturbance of the ocean floor, generated principally by a shallow submarine earthquake, but also by submarine earth movement, subsidence, or volcanic eruption". This isolated pulse of disturbance travels in liquid medium from one place to an other, but only in horizontal plane the dynamic characteristics will determine a constant speed. The pulse will continue to travel in this way until it reaches the solid floor at which places reflection processes of some sort will occur. So long that the water' depth is constant or greater the periodically repeated disturbance will appear also at the water surface. Sketches of this traveling pulse for t=0 and x=0 are shown in Fig. 1. The peak of this pulse will travel radial as a transverse wave in each horizontal plane inclusively at the ocean surface. The Actual dynamics of traveling pulse can be constructed by considering the equality of the traverse forces, due to the curvature of the wave in the y0z plane, with the restoring forces. Phenomenological we shall suppose further that the ocean's floor is disturbed and at a some instant let the configuration of some portion of water spatial column be as in Fig 1. Thus, for any elemental volume of water the net force acting on it will have the components in vertical and horizontal plane connected by the relation: $F_z = F_r tg\alpha = F_r \frac{dz}{dr}$.

Assuming that the traverse displacement is small so that α are small angles, in the, zx, plane the elementary transverse force have the components:

$$dF_z(\alpha_x) = \rho ghdA \frac{\partial}{\partial x}\frac{\partial z}{\partial x}dx = \rho ghdzdy \frac{\partial^2 z}{\partial x^2}dx$$

and similar in zy plane $dF_z(\alpha_y) = \rho gh\, dz\, dx \frac{\partial^2 z}{\partial y^2}dy$

The total force is the sum of these and the mass being accelerated is *dm=ρdxdydz*. Hence the equation of motion, $adm=dF_z$, becomes:

$$\frac{\partial^2 z}{\partial t^2} = gh\left(\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2}\right), \quad (2)$$

or in cylindrical symmetry, where $x^2+y^2=r^2$ we have

$$\frac{\partial^2 z}{\partial t^2} = gh\left(\frac{\partial^2 z}{\partial r^2} + \frac{1}{r}\frac{\partial z}{\partial r}\right) \quad (3)$$

The traveling waves, that represent solutions of the equation (3), are expanding circular wave fronts. Intuitively we can recognize that the amplitude of the crests becomes less as r increases because the disturbance's energy is being spread over the cylinders surfaces of increasing radius. In cylindrical symmetry, at the distance, ***r=r(t),*** of traveling wave front the elongation ***Ψ(r,t)=z*** is the momentary local displacement of water bodily. The transverse circular traveling wave front, that represent solutions of this differential equation in no viscous and incompressible idealized water, [4,5], have an amplitude that falls off, for constant depth, as $r^{-1/2}$. Circular waves, faraway from the source became in effect straight or plane waves. Thus is evident that, at large distance r, it can ignore the second term of the right side and the equations of transverse motion will be the well known differential equation of waves:

$$\frac{\partial^2 \Psi}{\partial t^2} = v^2 \frac{\partial^2 \Psi}{\partial x^2} \quad (4)$$

where $v^2=gh$ is a constant of liquid medium in the horizontal plane of source. This wave motion, visible only as the shallow water waves is a global traverse motion of the water found above to the disturbed ocean floor. However, because geophysical forces generate them, they carry geophysical energy and momentum. At the great levels, which can be devastating to mere humans these waves are named tsunamis. It is important to note that tsunamis have nothing to do with the tides or tidal wave, which are driven by the gravitational forces between the Earth and the Moon. In this sense the tsunamis is one of natural modes of vibration of the Earth-Ocean system. It is clear from the last equations that *gh* has the dimension of the square of velocity, and this will prove to be no one other than the speed of pulse circular waves which travel at the surface of the water at a given value of depth, *h*. It is easy to see that the differential equation of transverse geophysics wave motion describes the tsunamis waves and all the long waves of the open sea. The speed of wave front, named the phase velocity, will be:

$$v = \sqrt{gh} = \sqrt{\frac{p}{\rho}} \tag{5}$$

where p is the pressure at the maximum depth, h, touched by perturbation. The visible wavelength, λ, defined as the traveling path of front wave in one period, T, will be of the forms:

$$\lambda = vT = \sqrt{gh}\,2\pi\sqrt{\frac{h}{g}} = 2\pi h \tag{6}$$

Because the speed with which the pulse travel is the phase velocity many autors sad that tsunamis are nondispersive waves. This means that, in the tsunamis case where we folow the motion of an solitar wave crest, the group velocity is the same to the speed of frontwaves.The wavelength of tsunamis being longer that the depth we consider that seawater is nondispersive medium and the transport of energy in a tsunami wave take place at the group velocity of $(gh)^{1/2}$

4. Evaluated characteristics of "26th December 2004 Sumatra Tsunamis"

After a careful analysis of involved problem, we prove our phenomenological model by calculating the value of the catastrophic tsunamis wave caused by the Evaluated characteristics are in god accord with those of catastrophic tsunamis caused by the Indonesian earthquake of Richter magnitude greater than 9 and epicenter depth of less than 40 km under the water column of height $h=5\ km$. Hence, using the above relationships and values we obtain:

-period $T = 2\pi\sqrt{\frac{h}{g}} = 2\pi\sqrt{\frac{5000}{9,8}} = 141,9\,sec. = 2\,min.21,9\,sec.$

-freqvency $\nu = \frac{1}{T} = 7 \cdot 10^{-3}\,Hz$;

-angular frequency (pulsation) $\omega = \frac{2\pi}{T} = \sqrt{\frac{g}{h}} = \sqrt{\frac{9,8}{5000}} = 0,0443 rad / sec = 2,54 \, deg/ sec$

this is,in fact, the angular velocity of the water particles with which these roling in vertical plane;

in the sense of traveling on the wave crest and in oposite sense in the trench

-speed $v = \sqrt{gh} = \sqrt{9,8 \cdot 5000} = 221 m/s = 797\, km/h = 430 knot$;

-wavelength $\lambda = 2\pi h = 2\pi\, 5000 = 31,4 \cdot 10^3 m = 31,4 km = 16.95 nautmi$.

The calculated charcteristcs are in good accord with those observed and comunicated.

5. Discutions and conclusions

Without to discus all the energetic aspects of tsunamis our phenomenological model based on the concept of gravific wave can explain the different phenomena that appear in tsunamis waves propagation. They appear only in the case of a large displacement in the ocean floor caused by a small part of earthquake energy that is transited to seawater. . The observed amplitudes of tsunamis are not higher than one meter as they propagate in the deep ocean. They can grow to several meters by the time they reach shore. Near shore submarine landslides of millions of cubic meters of material can generate local damaging tsunamis waves that reached 10 meters and 120 km/h The hydrostatic pressure in the volume of seawater determines all the undulatory characteristics of all kind of seawater waves. This shallow water waves is a global transverse motion of the water lain above to the disturbed ocean floor that follows, with a smaller speed, the compressional seismic elastic waves. However, because geophysical forces generate them, they carry geophysical energy and momentum. At the great levels, which can be devastating to mere humans these waves are named, simple, tsunamis. It is important to note that these distant waves have nothing to do with the tides or tidal wave, which are driven by the gravitational forces between Earth and Moon. Tsunamis appear only as a result of a major landslide disturbance in seawater. In this sense they are one of natural modes of vibration of the Earth-Ocean system.

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