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# COUPLINGS AMONG PAULI-FIERZ THEORY AND AN ABELIAN THREE-FORM IN D=11*

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**Abstract**
The final goal of our research is to prove the uniqueness of the simple SUGRA in D=*11* because supersymmetry seems to be crucial in the attempts to reconcile quantum mechanics and gravitation. In D=*4* the uniqueness of the simple SUGRA was already proved in the framework of the BRST formalism [1].

## 1. Introduction

It is well known that the field spectrum of *D=11*, *N=1* SUGRA consists in a massless spin-2 field, a massless spin-3/2 field and a 3-form gauge field. In the free limit the action of simple SUGRA in *D=11* reduces to the sum between Pauli-Fierz, Rarita-Schwinger, and a standard abelian 3-form actions

$$S_0^L[h_{\mu\nu},\psi_\mu,A_{\mu\nu\rho}] = S_0^{PF}[h_{\mu\nu}] + S_0^{RS}[\psi_\mu] + S_0^{3F}[A_{\mu\nu\rho}]. \tag{1}$$

In order to determine the consistent interactions that can be added to action (1) we must study, beside the self-interactions, which are known from the literature, also the cross-couplings. The latter problem can be solved in two steps: firstly, we determine the interaction vertices containing only two of the three types of fields, and then the vertices including all the three kinds.

In this talk we present one of the ingredients mentioned in the above, namely the problem of constructing consistent interactions among the Pauli -Fierz and an abelian three-form gauge fields. We investigate these cross-couplings in the framework of the deformation theory [2] based on local BRST cohomology [3].

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**Free model**

Our starting point is the Lagrangian action represented by the sum between Pauli-Fierz and an abelian three-form action in eleven space-time dimensions

$$\begin{aligned} S_0^L[h_{\mu\nu}, A_{\mu\nu\rho}] &= S_o^{PF}[h_{\mu\nu}] + S_0^{3F}[A_{\mu\nu\rho}] = \\ &= \int d^{11}x[-\frac{1}{2}(\partial_\mu h_{\nu\rho})(\partial^\mu h^{\nu\rho}) + (\partial_\mu h^{\mu\rho})(\partial^\nu h_{\nu\rho}) - \\ &-(\partial_\mu h)(\partial_\nu h^{\nu\mu}) + \frac{1}{2}(\partial_\mu h)(\partial^\mu h) - \frac{1}{2\cdot 4!}F_{\mu\nu\rho\lambda}F^{\mu\nu\rho\lambda}]. \end{aligned} \tag{2}$$

The theory (2) is invariant under the gauge transformations

$$\delta_\varepsilon h_{\mu\nu} = \partial_{(\mu\varepsilon\nu)}, \delta_\varepsilon A_{\mu\nu\rho} = \partial_{[\mu\varepsilon\nu\rho]}. \tag{3}$$

The gauge parameters $\varepsilon_\mu$ and $\varepsilon^{\mu\nu}$ are bosonic, the last set being completely antisymmetric. The gauge algebra of the free theory (2) is Abelian.

We observe that if in (3) we make the transformations

$$\varepsilon_{\mu\nu} \to \varepsilon_{\mu\nu}^{(\theta)} = \partial_{[\mu\theta\nu]}, \tag{4}$$

then the gauge variation of the 3-form identically vanishes

$$\delta_{\varepsilon(e)} A_{\mu\nu\rho} = 0. \tag{5}$$

Moreover, if in (4) we perform the changes

$$\theta_\mu \to \theta_\mu^{(\phi)} = \partial_\mu \phi, \tag{6}$$

with $\phi$ an arbitrary scalar field, then the transformed gauge parameters (4) identically vanish

$$\varepsilon_{\mu\nu}{}^{\theta^\phi} = 0 \tag{7}$$

Meanwhile, there is no non-vanishing local transformation of $\phi$ that annihilates $\theta_\mu^{(\phi)}$ of the form (6), and hence no further local reducibility identity. All these allow us to conclude that the generating set of gauge transformations (3) is off-shell second-stage reducible.

The structure of the reducibility relations is important from the point of view of the BRST symmetry as it requires the introduction of a tower of ghosts for ghosts, as well as of their antifields.

**Construction of consistent interactions**

Due to the fact that the solution to the master equation contains all the information on the gauge structure of a given theory, we can reformulate the problem of introducing consistent interactions as a deformation problem of the solution to the master equation corresponding to the ``free" theory. If an interacting gauge theory can be consistently constructed, then the solution $S$ to the master equation associated with the ``free" theory can be deformed into a solution $\bar{S}$

$$\begin{aligned} S \to \bar{S} &= S + \lambda S_1 + \lambda^2 S_2 + ... \\ &= S + \lambda \int d^D x a + \lambda^2 \int d^D x b + ..., \end{aligned} \tag{8}$$

of the master equation for the deformed theory

$$(\bar{S}, \bar{S}) = 0, \tag{9}$$

such that both the ghost and antifield spectra of the initial theory are preserved. The equation (9) splits, according to the various orders in $g$, into

$$(S, S) = 0, \tag{10}$$

$$2(S_1, S) = 0, \tag{11}$$

$$2(S_2, S) + (S_1, S_1) = 0, \tag{12}$$

$$(S_3, S) + (S_1, S_2) = 0, \tag{13}$$

$$....$$

The equation (10) is fulfilled by hypothesis. The next one requires that the first-order deformation of the solution to the master equation, $S_1$, is a co-cycle of the ``free" BRST differential. However, only cohomologically non-trivial solutions to (11) should be taken into account, as the BRST-exact ones (BRST co-boundaries) correspond to trivial interactions. This means that $S_1$ pertains to the ghost number zero cohomological space of $s, H^0(s)$, which is generically nonempty due to its isomorphism to the space of physical observables of the ``free" theory. It has been shown (on behalf of the triviality of the antibracket map in the cohomology of the BRST differential) that there are no obstructions in finding solutions to the remaining equations ((12-13), etc.). However, the resulting interactions may be nonlocal, and there might even appear obstructions if one insists on their locality. The analysis of these obstructions can be done with the help of cohomological techniques.

For our free model the solution to the master equation reads as

$$\begin{aligned} S = S_0[h_{\mu\nu}, A_{\mu\nu\rho}] + \int d^{11}x (h^{*\mu\nu} \partial_{(\mu\eta\nu)} + A^{*\mu\nu\rho} \partial_{[\mu C_\nu \rho]} \\ + C^{*\mu\nu} \partial_{[\mu C_\nu]} + C^{*\mu} \partial_\mu C). \end{aligned} \tag{14}$$

**Main results**

By direct computation we obtain the first-order deformation in the interacting sector like

$$\begin{aligned}
S_1^{(\text{int})} &= \int d^{11}x\{-kC^*(\partial^\mu C)\eta_\mu - \frac{k}{2}C_\mu^*[C_\nu\partial^{[\mu\eta\nu]} - (\partial_\nu C)h^{\mu\nu} \\
&+ 2(\partial^\nu C^\mu)\eta_\nu] + kC_{\mu\nu}^*[h_\rho^\mu\partial^\rho C^\nu - (\partial^\rho C^{\mu\nu})\eta_\rho - \frac{1}{2}C_\rho\partial[\mu h\nu]\rho \\
&+ C^\nu{}_\rho\partial^{[\mu\eta\rho]}] - kA_{\mu\nu\rho}^*[\eta\lambda\partial^\lambda A^{\mu\nu\rho} + \frac{3}{2}A^{\nu\rho}{}_\lambda\partial[\mu\eta\lambda] - \frac{3}{2}(\partial^\lambda C^{\nu\rho})h_\lambda^\mu \\
&- \frac{3}{2}C^{\rho\lambda}\partial^{[\mu}h_\lambda{}^{\nu]}] + \frac{k}{4}F_{\mu\nu\rho\lambda}[\partial^\mu(A^{\nu\rho\mu}h_\sigma^\lambda) + \frac{1}{4!}F^{\mu\nu\rho\lambda}h \\
&- \frac{1}{3}F^{\mu\nu\rho\sigma}h_\sigma^\lambda] + p\varepsilon^{\mu_1\ldots\mu_{11}}A_{\mu_1\mu_2\mu_3}F_{\mu_4\ldots\mu_7}F_{\mu_8\ldots\mu_{11}}\},
\end{aligned} \tag{15}$$

where $k$ and p are arbitrary real constants. The consistency of the first-order deformation (the existence of the second-order deformation) requires that the real constant $k$ satisfies the equation

$$k(k+1) = 0, \tag{16}$$

with the non-trivial solution

$$k = -1. \tag{17}$$

The previous results can be summarized in the following theorem.

**Theorem**

*Under the assumptions of: i) space-time locality, ii) smoothness of the deformations in the coupling constant, iii) (background) Lorentz invariance, iv) Poincaré invariance (i.e. we do not allow explicit dependence on the space-time coordinates), v) the maximum number of derivatives in the interacting Lagrangian is two, the only consistent deformation of (2) involving a spin-2 field and an abelian 3-form gauge field reads as*

$$\begin{aligned}
&S^L[h_{\mu\nu}, A_{\mu\nu\rho}] = S_0^{PF}[h_{\mu\nu}] + S_0^{3F}[A_{\mu\nu\rho}] + \\
&+ \lambda\int d^{11}x[L_E - \frac{1}{4*4!}F_{\mu\nu\rho\lambda}F^{\mu\nu\rho\lambda}h + \\
&\frac{1}{2*3!}F_{\mu\nu\rho\lambda}F^{\mu\nu\rho\sigma}h_\sigma^\lambda - \frac{1}{4}F_{\mu\nu\rho\lambda}\partial^\mu(A^{\nu\rho\sigma}h_\sigma^\lambda) + \\
&+ q\varepsilon^{\mu_1\ldots\mu_{11}}A_{\mu_1\mu_2\mu_3}F_{\mu_4\ldots\mu_7}F_{\mu_8\ldots\mu_{11}}] + O(\lambda^2),
\end{aligned} \tag{18}$$

*and it is invariant under the gauge transformations*

$$\bar{\delta}_{\varepsilon} h_{\mu\nu} = \partial_{(\mu}\varepsilon_{\nu)} + \lambda\left(\frac{1}{2}(h_{\rho(\mu}\partial_{\nu)})\varepsilon^{p} - \varepsilon^{\rho}\partial_{(\mu}h_{\nu)\rho} + \varepsilon^{\rho}\partial_{\rho}h_{\mu\nu}\right) + O(\lambda^2), \quad (19)$$

$$\begin{aligned}\bar{\delta}_{\varepsilon,\varepsilon} A_{\mu\nu\rho} &= \partial_{[\mu}\varepsilon_{\nu\rho]} + \lambda[\varepsilon^{\lambda}\partial_{\lambda}A_{\mu\nu\rho} + \frac{1}{2}A^{\lambda}{}_{[\mu\nu}\delta^{\sigma}_{\rho]}\partial_{[\sigma\varepsilon\lambda]} - \\ &- \frac{1}{2}(\partial^{\lambda}\varepsilon_{[\mu\nu})h_{\rho]\lambda} + \frac{1}{2}\varepsilon^{\lambda}{}_{[\mu}\partial_{\nu}h_{\rho]\lambda}] + O(\lambda^2),\end{aligned} \quad (20)$$

*which remain second-order reducible, with the first-order reducibility given by*

$$\varepsilon_{\mu\nu} \to \varepsilon^{(\theta)}_{\mu\nu} = \partial_{[\mu}\theta_{\nu]} + \frac{\lambda}{2}[(\partial^{\rho}\theta_{[\mu})h_{\nu]\rho} + \theta^{\rho}\partial_{[\mu}h_{\nu]\rho}] + O(\lambda^2), \quad (21)$$

*and the second-order redundancy expresses by*

$$\theta_{\mu} \to \theta^{(\phi)}_{\mu} = \partial_{\mu}\phi - \frac{\lambda}{2}h_{\mu\nu}\partial^{\nu}\phi + O(\lambda^2). \quad (22)$$

In the formula (18) $L_E$ represents the cubic vertex of the Einstein-Hilbert Lagrangian.

Now, we make the connection between our results and those predicted by GR. It is known that in GR the Lagrangian action for a three-form interacting with a gravitational field is of the form

$$\begin{aligned}S[g_{\mu\nu}, \bar{A}_{\mu\nu\rho}] &= \int d^{11}\sqrt{-g}[\frac{2}{k^2}(R - 2\Lambda) - \\ &- \frac{1}{2*4!}g^{\mu\alpha}g^{\nu\beta}g^{\rho\gamma}g^{\lambda\delta}\bar{F}_{\mu\nu\rho\lambda}\bar{F}_{\alpha\beta\gamma\delta}],\end{aligned} \quad (23)$$

being subject under the second-stage reducible gauge transformations

$$\frac{1}{k}\bar{\delta}_{\bar{\varepsilon}} g_{\mu\nu} = \bar{\varepsilon}_{\mu;\nu} + \bar{\varepsilon}_{\nu;\mu}, \quad (24)$$

$$\bar{\delta}_{\bar{\varepsilon},\bar{\varepsilon}}\bar{A}_{\mu\nu\rho} = \partial_{[\mu}\bar{\varepsilon}_{\nu\rho]} + \varepsilon^{-\lambda}\partial_{\lambda}\bar{A}_{\mu\nu\rho} + \bar{A}_{\sigma[\mu\nu}\partial_{\rho]}\varepsilon^{-\sigma}. \quad (25)$$

We observe that if in (25) we make the transformations

$$\bar{\varepsilon}_{\mu\nu} \to \bar{\varepsilon}^{(\theta)}_{\mu\nu} = \partial_{[\mu}\bar{\theta}_{\nu]}, \quad (26)$$

then the gauge variation of the 3-form identically vanishes

$$\bar{\delta}_{\varepsilon(e)}A_{\mu\nu\rho} = 0. \quad (27)$$

Moreover, if in (26) we perform the changes

$$\bar{\theta}_{\mu} \to \bar{\theta}^{(\bar{\phi})}_{\mu} = \partial_{\mu}\bar{\phi}, \quad (28)$$

with $\bar{\phi}$ an arbitrary scalar field, then the transformed gauge parameters (26) identically vanish

$$\bar{\varepsilon}_{\mu\nu}^{\left(\bar{\theta}^{(\phi)}\right)}0 \tag{29}$$

If we make the developments around the flat metric

$$g_{\mu\nu} = \sigma_{\mu\nu} + kh_{\mu\nu},$$

$$\bar{A}_{\mu\nu\rho} = A_{\mu\nu\rho} + \frac{k}{2} h^{\sigma}_{[\mu} A_{\nu\rho]\sigma} + O(k^2),$$

$$\bar{\varepsilon}^{\mu} = \varepsilon^{\mu} - \frac{k}{2} h^{\mu}_{\nu} \varepsilon^{\nu} + O(k^2),$$

$$\bar{\varepsilon}_{\mu\nu} = \varepsilon_{\mu\nu} + \frac{k}{2} h^{\sigma}_{[\mu} \varepsilon_{\nu]\sigma} + O(k^2),$$

$$\bar{\theta}_{\mu} = \theta_{\mu} + \frac{k}{2} h^{\sigma}_{\mu} \theta_{\sigma} + O(k^2),$$

$$\bar{\phi} = \phi,$$

then the leading order in the coupling constant $k$ of (23)-(29) is in agreement with our formulas (18)-(22).

**References**


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