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EQUILUBRUM AND TRANSPORT PROPERTIES OF THE AHARONOV BOHM RING WITH ATTACHED LEADS[1]

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Abstract
The deals with equilibrium and transport properties characterizing an Aharonov-Bohm ring with attached leads. This concerns the persistent current in the ring and the conductance, respectively. The dependence on the magnetic flux as well as on the gate voltage is discussed in both cases.
Keywords: Aharonov-Bohm rings, persistent currents, conductance.

Recent progress in electron beam lithography, ephitaxial growth and colloidal synthesis made possible fabrication and manipulation of semiconductor structures in which the carriers are confined in all three dimensions to a nanometer region. A quantum dot is a semiconductor crystal whose size is of the order of a few nanometers to few hundred nanometer [1,2]. Quantum dots confine electrons, holes, or excitons to a region on the order of the electron's de Broglie wavelength. The electrons on an 1D ring shaped nanowire which is threaded by a magnetic flux i.e. the Aharonov-Bohm (AB) ring, has received much interest [3-7]. The nonvanishing angular momentum of quantum ring in the presence of a magnetic field is closely related to the so-called persistent currents. Such currents have been observed in experiments with GaAs/GaAlAs rings [8-11]. Signatures of the Fano resonance [12] originating from the interference between a discrete energy level and a continuum, have been reported [13].

A further interesting configuration is done by a discretized Aharonov-Bohm ring with N_s sites, now with two attached semi-infinite leads [14]. These leads are attached to the sites 1 and n, as shown in Fig. 1. The point-like couplings between the ring and the leads are

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characterized by the hopping amplitudes t_R and t_L, where the subscripts "R" and "L" stand for "right" and "left", respectively. A point-like coupling between the leads, such as expressed by the hopping parameter t_c will also be accounted for. This latter coupling provides the continuous path for inter-lead electron transmission. In addition, there are tunneling effects of the electrons through the ring, which serves as a "discrete" path for the transmission just mentioned above.

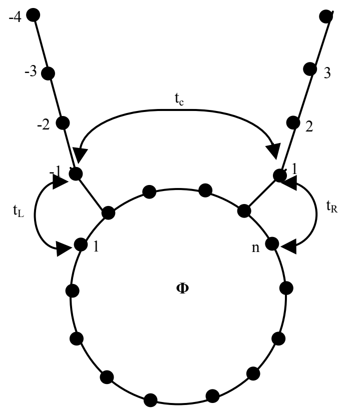

Fig. 1. Schematic view of the quantum ring threaded by a magnetic flux and attached to two current leads.

In order to proceed further let us denote the creation (annihilation) operator of the spinless electron on the ring, on the left and right leads by $c_l^+(c_l)$, $a_m^+(a_m)$ and $b_m^+(b_m)$, respectively. The site-numbers are given by *l=1,2,…N*$_s$, for the ring, *m = -1,-2,…*, for the left-lead, and *m = 1,2,…*, for the right lead. In order to describe the non-interacting leads, i.e. the pertinent 1D conductors, we shall resort to 1D tight-binding models with NN-interaction, in which case t_0 stands for the inherent hopping parameter.

The electron on the ring is described by the Hamiltonian

$$H_{Ring} = \sum_{l=1}^{N_S} (\varepsilon_l c_l^+ c_l + t_r \exp(i\varphi_R) c_l^+ c_{l+1} + t_r \exp(-i\varphi_R) c_{l+1}^+ c_l \qquad (1)$$

where the site energy is given by ε_l and t_r denotes the related hopping parameter. One realizes that (1) proceeds in a close analogy with the hopping Hamiltonian of a 2D lattice under the influence of a magnetic field

$$H_H = \sum_{l=1}^{N_s} (t_{ij} c_j^+ c_{ij} \exp(i\theta_{ij}) + H.c. \qquad (2)$$

where $\theta_{ij} = \frac{2\pi}{\phi_0} \int_i^j \vec{A} \cdot \vec{dl}$ is magnetic phase factor, $\vec{A}$ is the vector potential and $\phi_0 = hc/e$ is the magnetic flux quanta, as usual. Accordingly, the corresponding phase reads

$$\varphi_R = \frac{2\pi}{N} \frac{\phi}{\phi_0} \qquad (3)$$

The periodic boundary condition characterizing (1) reads $c_{l+N_S} = c_l$. Putting together both transmission channels leads to the tunneling Hamiltonian [14]

$$H_T = t_c a_{-1}^+ b_1 + t_L a_{-1}^+ c_1 + t_R b_1^+ c_n + t_c b_1^+ a_{-1} + t_L c_1^+ a_{-1} + t_R b_1 c_n^+ \quad (4)$$

which is responsible for the transport properties one looks for.

At this point we have to realize that a uniform gate voltage, say V_g, can be introduced via

$$\varepsilon_l = \varepsilon_l^0 + V_g \quad (5)$$

which provides the tuning-parameter for further investigations. It can be assumed, for convenience, that $\varepsilon_l^0 = 0$. It is understood that the spinless electron operators mentioned above satisfy usual canonical commutations relations like $\left[a_m, a_n^+\right] = \delta_{m,n}$ and similarly for b_m and c_l. It is also clear that

$$[a_m, b_n] = [a_m, c_l] = [b_n, c_l] = 0 \quad (6)$$

The present eigenvalue problem can be readily solved using the wavefunction ansatz

$$|\psi\rangle = \sum_{m \le -1} A_m a_m^+ |0\rangle + \sum_{m \ge 1} B_m b_m^+ |0\rangle + \sum_{l=1}^{N_S} C_l c_l^+ |0\rangle \quad (7)$$

so that we have to assume that $A_0 = B_0 = 0$. The electron amplitudes on the left-lead, on the ring and on the right-lead are denoted by A_m, C_l and B_m, respectively. Next, it can be easily verified that the stationary eigenvalue equation

$$H|\psi\rangle = E|\psi\rangle \quad (8)$$

where the total Hamiltonian reads

$$H = H_{Leads} + H_{Ring} + H_T \quad (9)$$

can be converted into coupled linear equations. These equations are given by

$$EA_m = t_0(A_{m+1} + A_{m-1}) + (t_c B_1 + t_L C_1)\delta_{m,-1} \quad (10)$$

if $m \le -1$,

$$EB_m = t_0(B_{m+1} + B_{m-1}) + (t_R C_1 + t_c A_{-1})\delta_{m,1} \quad (11)$$

if $m \ge 1$, and

$$(E - \varepsilon_l)C_l = t_r C_{l+1} \exp(i\varphi_R) + t_r C_{l-1} \exp(-i\varphi_R) + t_L A_{-1}\delta_{l,1} + t_R B_1 \delta_{l,n} \quad 12)$$

which yield typical manifestations in interaction and interaction-free regions, respectively.

Incoming and outgoing interaction-free regions in the leads can be identified via $m \le -2$ and $m \ge 2$, respectively. Within such interaction-free regions equations (10) and (13) exhibit plane wave solutions like

$$A_m = \exp(ik(m+1)) + r\exp(-ik(m+1)) \tag{13}$$

and

$$B_m = t\exp(ikm) \tag{14}$$

where k is the dimensionless wave number, while r and t denote reflection and transmission amplitudes. The energy of the incident electron is given by

$$E_{in} = 2t_0 \cos k \tag{15}$$

in terms of units for the lattice spacing is unity. Concerning the ring, the interaction-free regions are specified by $l \ne 1$ and $l \ne n$. This gives the equation

$$(E - \varepsilon_l)C_l = t_r C_{l+1}\exp(i\varphi_R) + t_r C_{l-1}\exp(-i\varphi_R) \tag{16}$$

in accord with (12). Invoking again plane-wave solutions like

$$C_l = \exp(i\tilde{k}l) \tag{17}$$

enables us to derive the ring energy as

$$E = E_r = \varepsilon_l + 2t_r\cos(\tilde{k} + \varphi_R) \tag{18}$$

where $\tilde{k} = \frac{2\pi n}{N_s}$. One would then have $\tilde{k} \in [0, 2\pi)$ if $n \in [0, N_s)$, which means that we deals with N_s - levels $(n = 0,1,2,..., N_s - 1)$. Restricting ourselves to l –values for which $l \ne 1$ and $l \ne n$, one finds "discretized" continuity equations like

$$\frac{\partial}{\partial t}\rho_l + \nabla I_l = 0 \tag{19}$$

by virtue of (16) For this purpose EC_l has been replaced, for the moment, by $i\hbar\partial C_l / \partial t$. This amounts to start from the time dependent Schrödinger-equation $H|\psi\rangle = i\hbar\partial|\psi\rangle / \partial t$ instead of (8). This time one deals with the left-hand discrete derivative

$$\nabla I_l = I_l - I_{l-1} \tag{20}$$

Accordingly, there is $\rho_l = -e|C_l|^2$ and

$$I_l = \frac{2e}{\hbar} t_r \operatorname{Im}(C_l^* C_{l+1}\exp(i\varphi_R)) \tag{21}$$

which plays the role of a particular contribution to the persistent current in the ring. So we have found a suitable description of the ring-current, which is provided specifically by the space discreteness. The total persistent current in the ring should then be given reasonably by the selected average

$$I = < I_l >' = \frac{1}{N_s - 2} \sum_{l \neq 1, l \neq n} I_l \tag{22}$$

in which the primed summation indicates that terms for which $l = 1$ and $l = n$ are ruled out. This corresponds to the implementation of a continuity equation like

$$\frac{\partial p}{\partial t} + \nabla I = 0 \tag{23}$$

where the corresponding charge density reads

$$\rho = \frac{1}{N_s - 2} \sum_{l \neq 1, l \neq n} \rho_l \tag{24}$$

Of a special interest is the transmission probability

$$T_p = |B_1|^2 \tag{25}$$

which yields the conductance by virtue of the well-known relationship (see e.g. [15] Datta (1995))

$$G = \frac{2e^2}{h} T_p \tag{26}$$

Next it can be easily verified, that one has

$$A_{-2} = A_{-1} \exp(ik) - 2i \sin k \tag{27}$$

and

$$B_2 = \exp(ik) B_1 \tag{28}$$

This yields the equations

$$\mathrm{A_{-1}(E - t_0 \exp(ik)) - t_c B_1 = -2it_0 \sin k + t_L C_1} \tag{29}$$

and

$$t_c A_{-1} - B_1 (E - t_0 \exp(ik) = -t_R C_n \tag{30}$$

Fixing the energy and accounting for (13) we have the opportunity to establish a number of $N_s + 2$ relationships concerning $C_l (l = 1, 2, ..., N_s), A_{-1}$ and B_1. We have to remark that the couplings can be expressed safely in units of t_0. What then remains is to solve numerically (13), (29) and (30), which results in interesting plots presented bellow. We shall in turn resort to the energy fixing $E = E_{in} = 0$ [14], which means in turn that $\exp(ik) = i$. The dependence of the conductance and of the persistent current on the gate voltage (magnetic flux) is displayed in Figs. 2 and 3 (Figs. 4 and 5), respectively. Units for which $t_0 = 1$ are used.

One sees that both conductance and persistent current are periodic functions of the magnetic flux with period Φ_0.

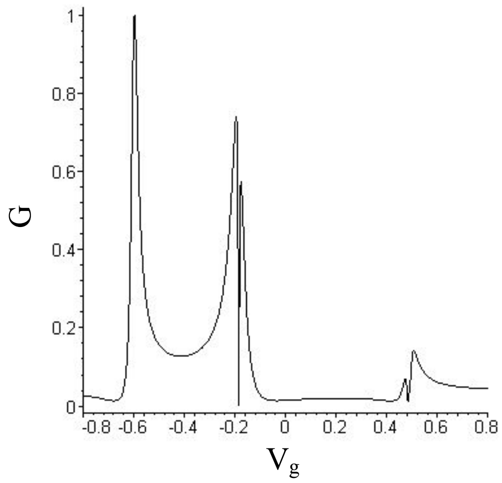

Fig. 2

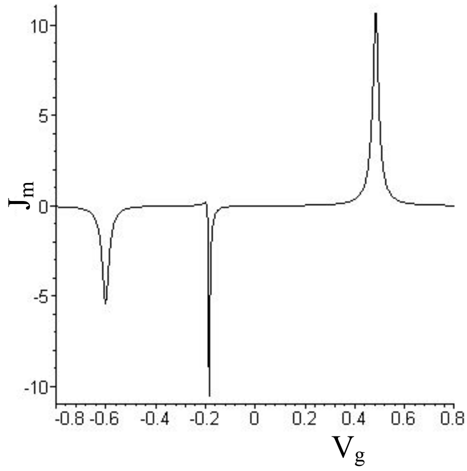

Fig.3

Fig.2 The V_g dependence of the conductance G for N_s =5, t_r=0.2, t_L=t_R=0.1, k=π/2 and $\phi = 0.3\phi_0$.
Fig. 3 The V_g dependence of the persistent current J_m for N_s =5, t_r=0.2, t_L=t_R=0.1, k=π/2 and $\phi = 0.3\phi_0$

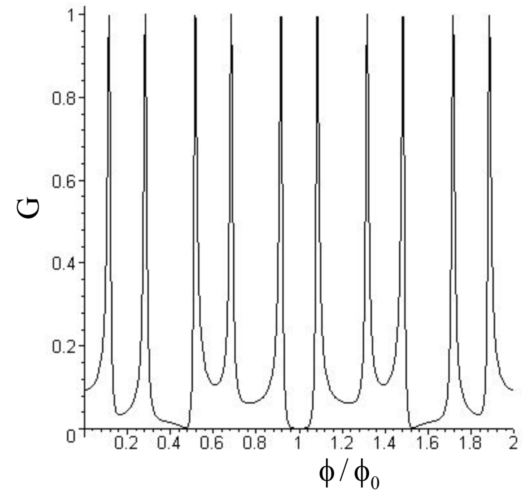

Fig.4

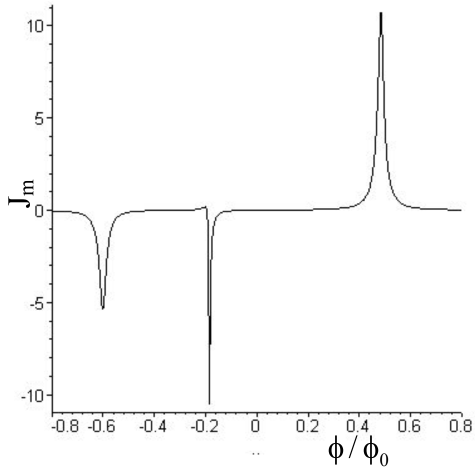

Fig.5

Fig.4 Conductance G versus adimensionally flux for Vg=0.03, N_s =5, t_r=0.2, t_L=t_R=0.1, k=π/2
Fig.5 Persistent current J_m versus adimensionally flux for Vg=0.03, N_s =5, t_r=0.2, t_L=t_R=0.1, k=π/2

In summary, we found that the dependence of the conductance and of the persistent current on the gate voltage can be studied in a rather controllable way. The peaks characterizing the dependence of he conductance on the gate voltage exhibit asymmetric Fano line-shapes, which are displayed in Fig.2. Such shapes are reminiscent to the interference effects between the continuous and discrete paths of the electron transmission mentioned above. The peaks exhibited by the persistent current versus the gate voltage rely on the levels of the ring, as indicated in Fig.3. Figure 4 shows that the dependence of the conductance on the magnetic flux is characterized by a periodic sequence of Fano-like profiles with the period given by the flux quantum. The same periodicity concerns the peaks of the persistent current displayed in Fig.5. These latter peaks can also be viewed as a manifestation of AB-oscillations. Without considering other details, we have to mention that electrons confined on a quantum Aharonov-Bohm ring are able to form a spin singlet state with electrons in the leads. This results in the implementation of a pronounced many-body Kondo-effect at lower fields, which received much attention during the last decade (see e.g. Keyser et. al. (2003)). However, Coulomb-blockade effects have also to be accounted for larger fields [16].

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